Benchmark: sensitivity analysis of a five bar mechanism

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Abstract

The present document describes the five bar mechanism and the simulation chosen as a benchmark problem for sensitivity analysis of multibody systems.

1 Description of the five bar mechanism

The case study chosen is the five-bar mechanism with 2 degrees of freedom shown in figure 1.

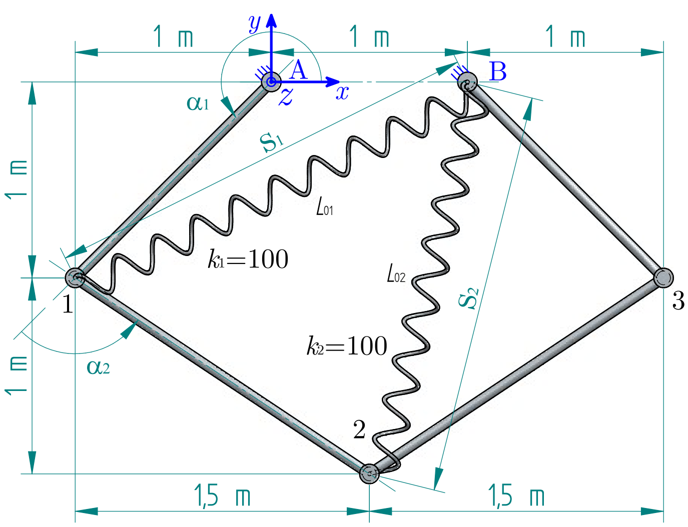

Figure 1: The five-bar mechanism

The five bars are constrained by five revolute joints located in points A (origin of global coordinates), 1, 2, 3 and B, with A and B attached to the ground.

The masses of the bars are $m_{A1} = 1$ kg, $m_{12} = 1.5$ kg, $m_{23} = 1.5$ kg, $m_{3B} = 1$ kg and the polar moments of inertia are calculated under the assumption of an uniform distribution of mass. The mechanism is subjected to the action of gravity, $\mathbf{g} = \begin{bmatrix} 0 & -9.81 & 0 \end{bmatrix}^{\mathrm{T}}$ m/s^2, and two elastic forces coming from the springs. The stiffness coefficients of the springs are $k_1 = k_2 = 100$ N/m and their natural lengths are chosen $L_{s1} = \sqrt{2^2 + 1^2} = \sqrt{5}$ m and $L_{s2} = \sqrt{2^2 + 0.5^2} = \sqrt{17}/2 = \sqrt{4.25}$ m, coincident with the initial configuration shown in Fig.1.

2 Five-bar mechanism: dynamic problem description and solution

The response of the system is shown in figures 2, 3 and 4 for a 5 seconds simulation starting from the configuration presented in figure 1 with zero initial velocities. Figure 2 represents the distances s_1, s_2 and their time derivatives, $\dot{s}_1$, $\dot{s}_2$, $\ddot{s}_1$, $\ddot{s}_2$; figure 3 shows the angles α_1, α_2 and time derivatives $\dot{\alpha}_1$, $\dot{\alpha}_2$, $\ddot{\alpha}_1$, $\ddot{\alpha}_2$; while figure 4 represents the position, velocity and acceleration of point 2 in the x-y plane.

The files *DynDistances.csv*, *DynAngles.csv*, *Dynp2.csv*, *Dyndp2dt.csv* and *Dynd2p2dt2.csv* contain the numerical results sampled every 10^{-2} s for checking.

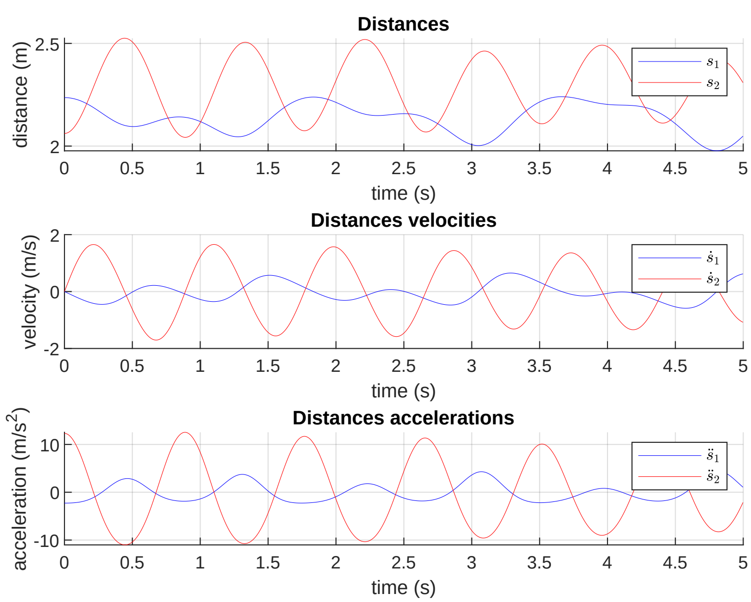

Figure 2: Dynamic response: distances s_1, s_2 (top); time derivatives $\dot{s}_1$, $\dot{s}_2$ (middle); second time derivatives $\ddot{s}_1$, $\ddot{s}_2$ (bottom).

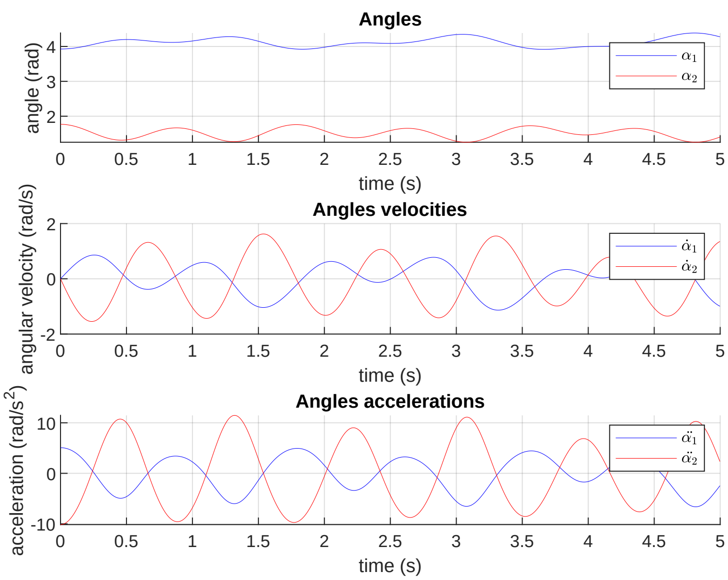

Figure 3: Dynamic response: angles α_1 and α_2 (top); time derivatives $\dot{\alpha}_1$, $\dot{\alpha}_2$ (middle); second time derivatives $\ddot{\alpha}_1$, $\ddot{\alpha}_2$ (bottom).

3 Five-bar mechanism: sensitivity problem description and solution

For the sensitivity analysis, the following array of objective functions is considered:

$$\boldsymbol{\psi} = \begin{bmatrix} \psi^1 \\ \psi^2 \\ \psi^3 \end{bmatrix} \tag{1}$$

with

$$\psi^1 = \int_{t_0}^{t_F} (\mathbf{r}_2 - \mathbf{r}_{20})^{\mathrm{T}} (\mathbf{r}_2 - \mathbf{r}_{20}) \,\mathrm{d}t = \int_{t_0}^{t_F} g_1 \mathrm{d}t \tag{2a}$$

$$\psi^2 = \int_{t_0}^{t_F} \dot{\mathbf{r}}_2^{\mathrm{T}} \dot{\mathbf{r}}_2 \mathrm{d}t = \int_{t_0}^{t_F} g_2 \mathrm{d}t \tag{2b}$$

$$\psi^3 = \int_{t_0}^{t_F} \ddot{\mathbf{r}}_2^{\mathrm{T}} \ddot{\mathbf{r}}_2 \mathrm{d}t = \int_{t_0}^{t_F} g_3 \mathrm{d}t \tag{2c}$$

As parameters to obtain the sensitivities, three different sets of parameters are going to be considered:

1. Parameters affecting the forces: with this set of parameters we consider the case in which the parameters affect the generalized forces vector, more

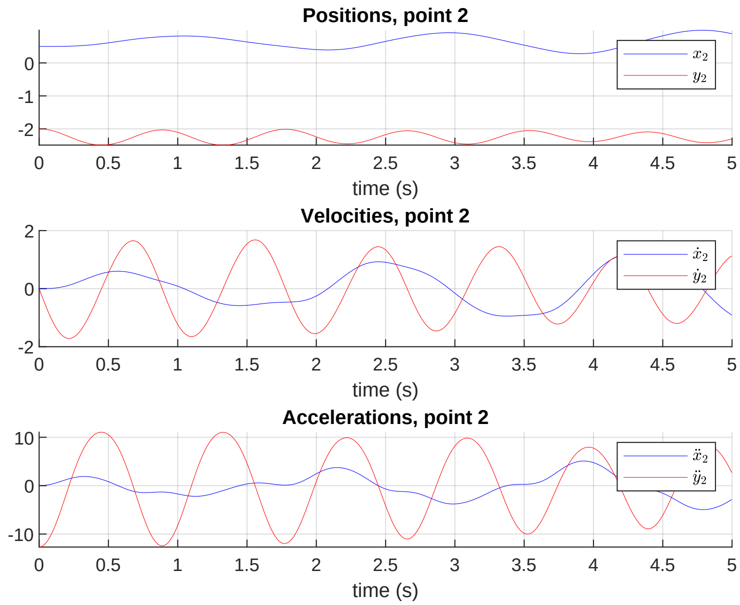

Figure 4: Dynamic response of point 2: positions (top); velocities (middle); accelerations (bottom).

specifically the natural lengths of the springs are going to be chosen as parameters, $\boldsymbol{\rho}_1 = \begin{bmatrix} L_{01} & L_{02} \end{bmatrix}^{\mathrm{T}}$.

2. Parameters affecting the geometry of masses: with this third set of parameters, we take into account the case in which the parameters affect to some components of the mass matrix (masses, center of masses or inertia tensors) and/or the gravity forces vector. In this benchmark example the mass of bar 1 and the longitudinal local coordinate of its center of mass are considered: $\boldsymbol{\rho}_2 = \begin{bmatrix} m_{A1} & \bar{x}^G_{A1} \end{bmatrix}^{\mathrm{T}}$.

3. Parameters affecting the geometry: with the second set of parameters, we explore the case in which the parameters affect the geometry of the system. In this case we will consider the length of bar A1, $\boldsymbol{\rho}_3 = \begin{bmatrix} L_{A1} \end{bmatrix}$.

Then, the vector of parameters for the sensitivity analysis is $\boldsymbol{\rho}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{\rho}_1^{\mathrm{T}} & \boldsymbol{\rho}_2^{\mathrm{T}} & \boldsymbol{\rho}_3^{\mathrm{T}} \end{bmatrix}$

The outcome of the sensitivity analysis is the gradient of the objective function (2), $\boldsymbol{\psi}' = \frac{\mathrm{d}\boldsymbol{\psi}}{\mathrm{d}\boldsymbol{\rho}}$.

The initial sensitivities of the degrees of freedom are chosen such that $[\alpha_1']_{t_0} = \left[\frac{\mathrm{d}\alpha_1}{\mathrm{d}\boldsymbol{\rho}}\right]_{t_0} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} = [\alpha_2']_{t_0} = \left[\frac{\mathrm{d}\alpha_2}{\mathrm{d}\boldsymbol{\rho}}\right]_{t_0}$.

The objective functions and their sensitivities are shown in figures 5 and 6 and the results are also included in the data files, *SensitObjfunc1.csv*, *SensitObjfunc2.csv* and *SensitObjfunc3.csv* sampled every 10^{-2} s.

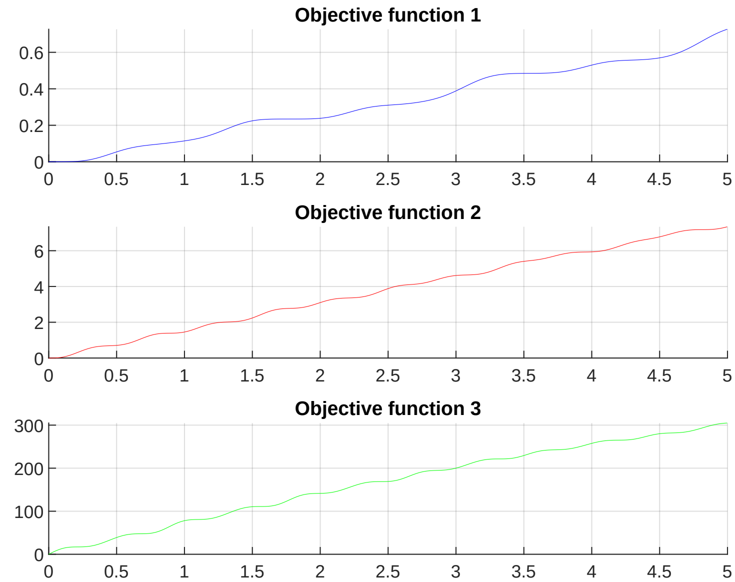

Figure 5: Objective functions over time $\int_{t_0}^{t} g_i \mathrm{d}t$ with $t \in [0, t_F]$, $i = 1, 2, 3$.

The reference solutions provided for the dynamics and sensitivity analysis have been obtained with a time step of 10^{-5} s using two different formulations [1] to guarantee convergence.

4 Benchmark error calculation

The final values of the sensitivities ($t = 5$ s) for the reference solution are included in table 1 along with the maximum allowed error for each sensitivity. The error of each solution should be evaluated per each objective function as the 2-norm of the difference between the solution and the reference:

$$error^{\boldsymbol{\psi}^i} = \left\| \left(\boldsymbol{\psi}^i\right)' - \left(\boldsymbol{\psi}^i\right)'_{Ref} \right\|_2, \quad i = 1, 2, 3. \tag{3}$$

Valid solutions should be below the maximum error tolerance for each objective function. The tolerances are included in the last column of Table 1:

$$error^{\boldsymbol{\psi}^i} \leq \epsilon_i, \quad i = 1, 2, 3. \tag{4}$$

[1] More specifically, the formulations described in [Dopico et al., 2015] and [Dopico et al., 2018] have been used.

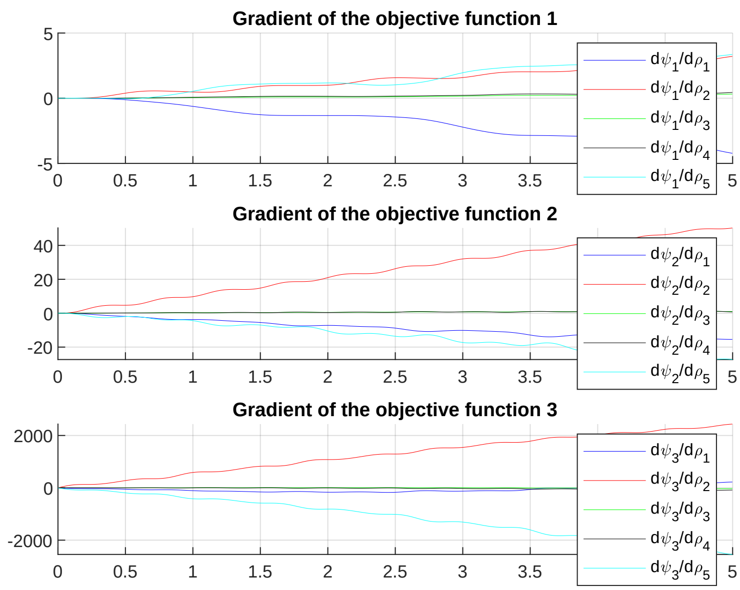

Figure 6: Gradient of the objective functions over time $\int_{t_0}^{t} \frac{\mathrm{d}g_i}{\mathrm{d}\boldsymbol{\rho}} \mathrm{d}t$ with $t \in [0, t_F]$, $i = 1, 2, 3$.

A single condition is required for publishing in the benchmark platform. For this purpose, the following single condition can be followed from conditions (4):

$$error^{\boldsymbol{\psi}} = \sum_{i=1}^{3} \frac{\epsilon_3}{\epsilon_i} error^{\boldsymbol{\psi}^i} \leq 3\epsilon_3 = 2.1 \tag{5}$$

Observe that condition (5) is necessary but not sufficient for (4) but it can be considered as a reasonable metric for the satisfaction of all the objective functions.

The file *results.txt*, attached to the solution, provides 3 rows with the sensitivities and the error (3), corresponding to each one of the objective functions.

References

[Dopico et al., 2018] Dopico, D., González, F., Luaces, A., Saura, M., and García-Vallejo, D. (2018). Direct sensitivity analysis of multibody systems with holonomic and nonholonomic constraints via an index-3 augmented lagrangian formulation with projections. *Nonlinear Dynamics*.

[Dopico et al., 2015] Dopico, D., Zhu, Y., Sandu, A., and Sandu, C. (2015). Direct and adjoint sensitivity analysis of ordinary differential equation multi-

	Reference solution	Maximum error tolerance (ϵ_i)
$\left(\boldsymbol{\psi}^1\right)'_{L_{s1}}$	-4.228811	
$\left(\boldsymbol{\psi}^1\right)'_{L_{s2}}$	3.211604	
$\left(\boldsymbol{\psi}^1\right)'_{m_{A1}}$	0.318657	1×10^{-3}
$\left(\boldsymbol{\psi}^1\right)'_{x^G_{A1}}$	0.442351	
$\left(\boldsymbol{\psi}^1\right)'_{L_{A1}}$	3.359810	
$\left(\boldsymbol{\psi}^2\right)'_{L_{s1}}$	-15.45207	
$\left(\boldsymbol{\psi}^2\right)'_{L_{s2}}$	50.30879	
$\left(\boldsymbol{\psi}^2\right)'_{m_{A1}}$	0.97012	1×10^{-2}
$\left(\boldsymbol{\psi}^2\right)'_{x^G_{A1}}$	0.74560	
$\left(\boldsymbol{\psi}^2\right)'_{L_{A1}}$	-27.35924	
$\left(\boldsymbol{\psi}^3\right)'_{L_{s1}}$	221.6400	
$\left(\boldsymbol{\psi}^3\right)'_{L_{s2}}$	2436.607	
$\left(\boldsymbol{\psi}^3\right)'_{m_{A1}}$	-32.4975	7×10^{-1}
$\left(\boldsymbol{\psi}^3\right)'_{x^G_{A1}}$	-85.6567	
$\left(\boldsymbol{\psi}^3\right)'_{L_{A1}}$	-2546.590	

Table 1: Reference solution for sensitivities at $t = 5$ s. and maximum allowed error.

body formulations. *Journal of Computational and Nonlinear Dynamics*, 10 (1)(1):1–8.